

# Real-Time Parameter Identification for Self-Designing Flight Control

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## Abstract

A self-designing flight control system (SDFCS) could provide a cost-effective means for developing controllers for new aircraft by eliminating analyst-intensive design of numerous individual controllers, each optimized for a single flight condition. Additionally, the SDFCS could improve the capabilities of existing aircraft by enhancing control performance in new flight regimes such as high angle-of-attack or post-stall maneuvers. Finally, the SDFCS could automatically reconfigure the control system to account for sudden changes such as may result from airframe and/or effector impairment(s). Rapid identification of time-varying, nonlinear plants is an important enabling technology for most SDFCS concepts. In this paper, the authors present a modified sequential least squares (MSLS) parameter identification method and compare its performance to that of standard RLS techniques using a simulated nonlinear F-16 with multi-axes thrust-vectoring (MATV) aircraft. It is shown that MSLS offers significant improvement in performance over conventional RLS parameter identification by providing: (1) a recursive estimation algorithm that penalizes noisy estimates and is less subject to ill-conditioning as its forgetting factor is reduced, (2) detection of airframe and effector impairments and corresponding adjustments of the algorithm settings, and (3) an intelligent supervisor that injects a minimum level of effector random activity to ensure identifiability.

## Introduction

This work is motivated by the need for rapid on-line identification of aircraft parameters that may be varying with time at unknown rates. The goal of the work outlined below is to create a parameter estimation algorithm that is: (a) less subject to numerical ill-conditioning, (b) less sensitive to noisy measurements, and (c) able to detect and track sudden changes in parameters that can arise if an airframe and/or effectors is/are impaired or if a low-order model is used to represent a higher-order system [1] [2].

In general, the linear parameter estimation problem may be stated as follows: find the "best" estimate of the parameter vector,  $\underline{\theta} \leftarrow^n$ , in the linear model:

$$y(n) = \underline{\theta}^T \underline{\phi}(n) + v(n) \quad (1)$$

where  $\underline{\phi} \leftarrow^n$  is a vector of known measurements,  $v(n)$  is the residual error due to measurement noise or unmodeled dynamics, and  $y(k)$  is the system output as measured at sample  $k$ . The most common way to find the "best" estimate is to find the parameter vector that minimizes a cost function,  $J$ , that is the sum of the squares of the residual,  $v(n)$ , over a set of  $k$  observations:

$$J = \sum_{n=1}^k [y(n) - \underline{\theta}^T \underline{\phi}(n)]^2 \quad (2)$$

This is the *equation-error* or *minimum-variance* estimate. For batch estimation problems, the cost function can be rewritten in matrix form as

$$J = [\underline{y} - \underline{\Delta} \underline{\theta}]^T [\underline{y} - \underline{\Delta} \underline{\theta}] \quad (3)$$

where  $\underline{y} = [y(1), y(2), \dots, y(k)]^T$  and

$$\underline{\Delta} = [\underline{\phi}(1), \underline{\phi}(2), \dots, \underline{\phi}(k)]^T.$$

The most computationally efficient batch parameter estimation routines are based on the *normal* equations [3]. If  $k \geq n$ , the normal equations may be used to find a unique set of parameters,  $\underline{\theta}^*$ , that minimizes (3):

$$\underline{\theta}^* = (\underline{\Delta}^T \underline{\Delta})^{-1} \underline{\Delta}^T \underline{y} \quad (4)$$

To allow Recursive Least Squares (RLS) algorithms to vary rapidly with time, historical information must be quickly forgotten. This is most often accomplished via the use of a forgetting factor,  $\lambda$ , which, in effect, introduces an exponentially decaying weight on prior measurements [4]. The reduction of  $\lambda$ , however, may cause the identification method to become numerically ill-conditioned as the matrix of observations,  $\underline{\Delta}$ , becomes singular due to the lack of sufficient information.

One problem with the normal equation solution is that if  $\underline{\Delta}$  is moderately ill-conditioned,  $\underline{\Delta}^T \underline{\Delta}$  may be severely degraded [3]. Most recursive parameter identification routines, including RLS and the Kalman-Bucy filter, depend upon keeping track of

$$\underline{P} = (\underline{\Delta}^T \underline{\Delta})^{-1} \quad (5)$$

which has the same numerical ill-conditioning problems described in the normal equations [5]. A number of techniques have been proposed to overcome the ill-

conditioning problem described above, including Bierman's U-D factorization algorithm [6] and Bobrow and Murray's Sequential Least Squares (SLS) algorithm [7]. For this work, the authors choose to investigate modifications to the SLS algorithm due to the ease with which additional constraints can be incorporated into it.

## Sequential Least Squares

The following is a summary of the SLS algorithm; a full derivation is presented in [7]. Given a system of equations of the form

$$\underline{y} = \underline{A} \underline{\theta} \quad (6)$$

find an orthogonal transformation,  $\underline{Q}$ , such that

$$\underline{Q} \underline{A} = \begin{bmatrix} \underline{R} \\ 0 \end{bmatrix} \quad (7)$$

where  $\underline{R}$  is an upper-triangular matrix. If we define

$$\underline{Q} \underline{y} = \begin{bmatrix} \bar{y} \\ \tilde{y} \end{bmatrix} \quad (8)$$

and apply  $\underline{Q}$  to both sides of (6), then the least-squares estimate of  $\underline{\theta}$  can be found by solving the system

$$\bar{y} = \underline{R} \underline{\theta}^* \quad (9)$$

where  $\underline{\theta}^*$  can be found by simple back-substitution because  $\underline{R}$  is triangular. For well-conditioned problems,  $\underline{\theta}^*$  is identical to the least-squares estimate found by solving the normal equations [4].

Given an existing  $\underline{R}$ , a new observation at time,  $n$ , may be incorporated into the system by appending the observation to (9) and re-arranging so that the system solves for the required change in the parameter vector,  $\Delta \underline{\theta}$ . Thus,

$$\begin{bmatrix} \underline{R} \\ \underline{\phi}(n) \end{bmatrix} \Delta \underline{\theta} = \begin{bmatrix} 0 \\ \varepsilon(n) \end{bmatrix} \quad (10)$$

in which  $\varepsilon(n)$  is the current prediction error defined by

$$\varepsilon(n) = y(n) - \underline{\theta}_{n-1}^T \underline{\phi}(n) \quad (11)$$

where  $\underline{\theta}_{n-1}^T$  represents the least-squares parameter estimate as computed during the previous update. For well-conditioned problems, this is equivalent to an RLS update, where at each sample period the distortion function,  $d()$ , to be minimized is

$$d() = \frac{1}{2} \varepsilon^2(n) \quad (12)$$

Once again, the system in (11) may be triangularized as described above using a different orthogonal matrix,  $\underline{Q}$ . In [7], a Givens rotation is used to zero out the  $\underline{\phi}(n)$  row vector on the left side of (10); however, any number of orthogonal transformations may be used, such as a Householder transformation.

To introduce a forgetting factor,  $\lambda$ , one need only pre-multiply  $\underline{R}$  by  $\lambda^{1/2}$  in (10) before the updating is performed. (Note that because SLS is a square-root algorithm, the square-root of the RLS forgetting factor,  $\lambda$ , should be used. Bobrow and Murray did not do this in their original paper, resulting in some misleading SLS/RLS comparisons).

## Constrained Cost Function

In addition to numerical conditioning problems, small forgetting factors also tend to result in parameter estimates that are extremely sensitive to measurement noise and therefore tend to fluctuate wildly. The authors dealt with this problem by modifying the distortion function (12) so as to discourage excessively large changes in parameter values from sample to sample. The modification chosen is the addition of a simple penalty on the magnitude of the parameter change,  $\Delta \underline{\theta}$ :

$$d() = \frac{1}{2} \varepsilon^2(n) + \Delta \underline{\theta}^T \underline{W}_0 \Delta \underline{\theta} \quad (13)$$

The above modified distortion function can be incorporated into the augmented system of equations (10) as follows:

$$\begin{bmatrix} \underline{R} \\ \underline{\phi}(n) \\ \underline{W}_0 \end{bmatrix} \Delta \underline{\theta} = \begin{bmatrix} 0 \\ \varepsilon(n) \\ 0 \end{bmatrix} \quad (14)$$

and (14) may be triangularized in the same manner described in the previous section. In general,  $\underline{W}_0$  may be selected to be a matrix with constant diagonal terms. If it is known *a priori* that some parameters are likely to change more rapidly than others, the corresponding  $\underline{W}_0$  values for those parameters may be made smaller.

Such a modification to the distortion function may seem counter-intuitive, because the goal of the modified algorithm is to track rapidly varying parameters; however, simulation results show that a small amount of penalty on parameter fluctuation will greatly attenuate parameter noise while maintaining a high degree of tracking ability.

It should be noted that the introduction of the additional penalty on parameter change is not equivalent to low-pass filtering the parameter estimates. In general, a low-pass filtering approach would introduce a lag in the

estimate track, whereas the penalty method does not, as is seen in the next section.

## Linear Simulation Results

As an example, consider the system

$$y(n) = [\theta_1, \theta_2] \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} + v(k) \quad (15)$$

where  $\theta_1$  is time-varying, and  $v(k)$  is Gaussian white noise with  $\sigma^2 = 0.1$ . To intentionally create a poorly conditioned observation matrix, the measurements  $\phi_1$  and  $\phi_2$ , which would correspond to state measurements and/or surface deflections in a flight-control context, were intentionally driven with highly correlated Gaussian signals:

$$\phi_1(n) = N(0, 1) \quad (16)$$

$$\phi_2(n) = \phi_1(n) + N(0, 0.01) \quad (17)$$

For this experiment,  $\theta_2$  was fixed, and the modified SLS parameter estimates for a time-varying  $\theta_1$  are shown in Fig. 1. Note that in the figure the dashed lines represent the true parameter values and the solid lines represent the parameter estimates. In Fig. 1, the two lines are difficult to distinguish due to the accuracy of the tracking algorithm.

For purposes of comparison, Fig. 2 shows the RLS estimate of the same system. Normally, to remove the noise from the estimate shown in Fig. 2, one would increase the forgetting factor,  $\lambda$ . However, as shown in Fig. 3, while this does smooth out the estimates while the parameter is not varying with time, it yields no improvement while  $\theta_1$  is time-varying.

It was found that the magnitude of the random signal in (17) could be halved resulting in only minor degradation in the MSLS estimates; however, any reduction from the value set in (17) resulted in a complete breakdown of the RLS algorithm.

## Abrupt Change Detection

While the constrained cost function allows tracking of parameters that change relatively rapidly with time, it does increase the time required to converge to a new value of a parameter that changes instantaneously. This behavior can be remedied by introducing a change detector and modifying the identification algorithm settings when abrupt changes are detected.

In previous work, Barron Associates, Inc. (BAI) successfully demonstrated the use of Polynomial Neural Networks (PNNs) as impairment detectors [1].

In [8], a detection method is presented that tracks a moving average of the square of the prediction error,  $\epsilon$ , as

computed in (11). An abrupt change is declared if the moving average exceeds a threshold. In the current work, the authors also use the value of the prediction error, but the statistic computed is a measure of the *per unit* increase in the magnitude of the prediction error over the time-averaged nominal value. It is found that using this technique, a single threshold value can be established that gives good detection performance for a variety of state equations and flight conditions.

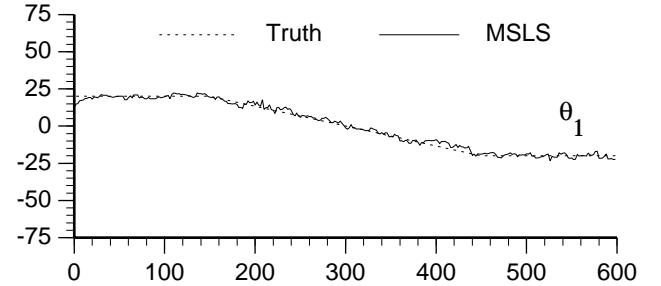


Fig. 1: MSLS Estimate of Time-Varying Parameter ( $\lambda = 0.1, \underline{W}_0 = 0.03 \underline{I}$ )

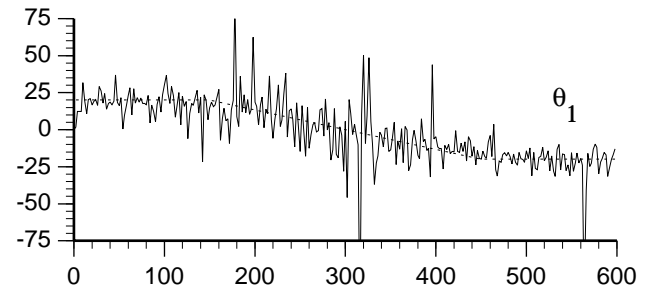


Fig. 2: RLS Estimate of Time-Varying Parameter ( $\lambda = 0.1$ )

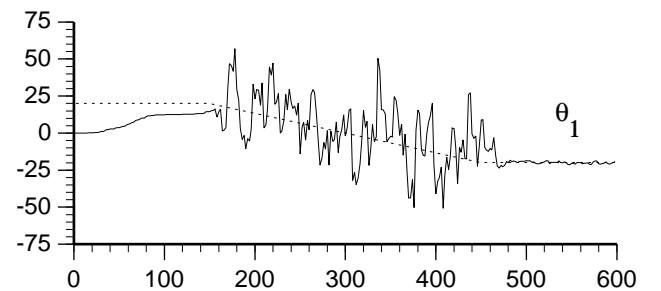


Fig. 3: RLS Estimate of Time-Varying Parameter ( $\lambda = 0.9$ )

Let  $\bar{\epsilon}_k$  represent a moving average, over some window of length  $N$ , of the absolute prediction errors up to but not including time  $k$ :

$$\bar{\epsilon}_k = \frac{1}{N} \sum_{n=k-N}^{k-1} |\epsilon(n)| \quad (18)$$

An abrupt change is declared if

$$\frac{\varepsilon(k) - \bar{\varepsilon}_k}{\bar{\varepsilon}_k} > T_\varepsilon \quad (19)$$

where  $T_\varepsilon$  is a user-specified threshold. Once a change is detected, all historical information (as contained in the  $\underline{R}$  matrix) is forgotten, and the values of  $\underline{W}_0$  and  $\lambda$  are reduced to encourage rapid reconvergence.

Figs. 4 and 5 show the results of MSLS on the same system described in (15) with and without the change detection. In this example,  $\theta_1$  was varied sinusoidally, and the value of  $\theta_2$  was changed abruptly at sample 300. Note the improved convergence rate in Fig. 4. For purposes of comparison, the RLS estimate of  $\theta_1$  is shown in Fig. 6; note that this is a more difficult tracking problem as can be seen by comparing Fig. 6 with Fig. 3.

Without change detection, both MSLS and RLS parameter estimates jump at the point of the change (Figs. 5 and 6); the peak values, however, are truncated to show the detail of the parameter tracking. For MSLS without change detection the parameter error at sample 300 was about 225; whereas for RLS the parameter error at sample 300 was almost 1,200.

## Active Noise Injection

The above examples illustrate the improvements that can be achieved by modifying a numerically stable parameter identification algorithm so that it uses a constrained cost function and can detect abrupt changes. While the inputs to the test problem were highly correlated; they provided sufficient excitation to allow MSLS to identify the system parameters.

In a flight control context, however, there are often circumstances in which the inputs are not varying sufficiently to allow proper identification. This problem may be addressed via the use of active noise injection (ANI). It is possible for the parameter identification process to inject small amounts of random activity into the effectors. This injection insures identifiability of the effector partials and assists in the identifiability of the state partials as well, depending on how the injected noise propagates through to the states.

Military flying qualities specifications allow perturbing signals provided the acceleration (presumably the acceleration felt by the pilot) due to the perturbing signals is less than 0.05 g rms [7]. Using this specification, one may use the identified effector partial derivatives to determine how additive noise will translate into acceleration felt by the pilot.

If the pilot is seated  $l_1$  feet in front of the aircraft center of mass along the body x-axis and  $l_2$  feet above the center of mass along the body z-axis, the square of the

acceleration experienced by the pilot,  $a_p$ , due to aircraft motion (not including gravity) may be calculated using the following relationship:

$$a_p^2 = \dot{u}^2 + (\dot{v} + l_2 \dot{P} + l_1 \dot{R})^2 + (\dot{w} - l_1 \dot{Q})^2 \quad (20)$$

where the angular rates are measured in rad./sec.

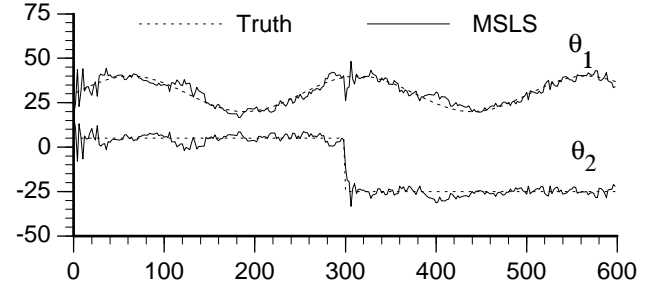


Fig. 4: MSLS with Abrupt Change Detection  
( $\lambda = 0.1, \underline{W}_0 = 0.03 \underline{I}$ )

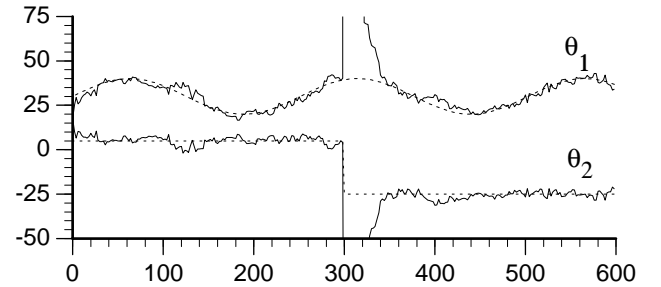


Fig. 5: MSLS without Abrupt Change Detection  
( $\lambda = 0.1, \underline{W}_0 = 0.03 \underline{I}$ )

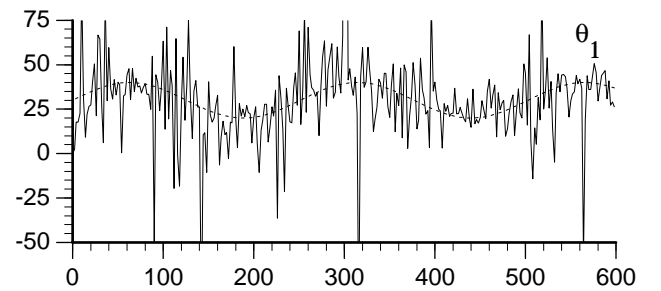


Fig. 6: RLS without Abrupt Change Detection  
( $\lambda = 0.1$ )

Assuming that a specified rms amount,  $\Delta$ , of active noise injection (ANI) is added to the displacement of each effector, the rms state accelerations,  $\underline{\ddot{x}}_{\Delta}$ , due to effector ANI can be estimated using the estimated effector partials contained in the  $\underline{B}$  matrix:

$$\dot{\underline{x}}_{\Delta} = |\hat{\underline{B}}| \underline{\Delta} \quad (21)$$

where  $\underline{\Delta}$  is a vector with  $\Delta$  in each element. Note that because rms values are used, the polarity of the actual effector deflection is uncertain; therefore, a "worst-case" scenario is considered by using the absolute value of the  $\underline{B}$  matrix. Using the same argument, the minus sign in (20) is changed to a plus sign when the equation is applied to rms values. If the order of the states in the state vector is

$$\underline{x} = [ \dot{u}, \dot{v}, \dot{w}, \dot{P}, \dot{Q}, \dot{R} ]^T \quad (22)$$

Eq. (18) becomes:

$$\begin{aligned} a_p^2 = & \Delta^2 [ (\sum b_{1j})^2 + \\ & (\sum b_{2j} + l_2 \sum b_{4j} + l_1 \sum b_{6j})^2 + \\ & (\sum b_{3j} + l_1 \sum b_{5j})^2 ] \end{aligned} \quad (23)$$

where  $\sum b_{ij}$  represents the sum of the absolute values of the elements in row  $i$  of the  $\underline{B}$  matrix. To allow for a 20% margin of error due to parameter estimation errors, the pilot acceleration due to ANI is never allowed to exceed 0.04 g rms. Additionally, this level of ANI is used only for a short period of time after an abrupt change is detected; at all other times, the level is set at 0.012 g rms. Additional logic is added to handle effector dynamics and to set limits on the ANI during periods in which the  $\underline{B}$  estimates are deemed unreliable; however, this logic is not discussed here.

## F-16/MATV Simulation Results

The MSLS algorithm, with all modifications described above, was incorporated into a nonlinear, time-varying, six-degree-of-freedom, F-16/MATV simulation [10]. Although the F-16 simulation was nonlinear, the parameter identification algorithm was tasked with identifying the terms in the  $\underline{A}$  and  $\underline{B}$  matrices of the linear state equations:

$$\dot{\underline{x}} = \underline{A} \underline{x} + \underline{B} \underline{\delta} \quad (24)$$

where

$$\underline{x} = [ u, v, w, P, Q, R, \Phi, \Theta, \Psi ]^T \quad (25)$$

$$\underline{\delta} = [ \delta_e, \delta_r, \delta_{dt}, \delta_{df}, \delta_r, \delta_{Te}, \delta_{Tr} ]^T \quad (26)$$

The maneuver selected for evaluation was a two-pound roll stick doublet executed during high angle-of-attack flight at 20,000 ft. and Mach 0.23. At one sec. into the maneuver, two lbs. of roll stick force were added to the trim value and maintained until five sec. into the maneuver; at five sec. two lbs. of roll stick force in the opposite direction were added to the trim value and maintained until nine sec. into the maneuver, at which

time the roll command was returned to trim. A pilot simulation module was used to keep  $\alpha$  at or near  $38.5^\circ$  during the course of the maneuver.

The MSLS algorithm was nominally set at  $\lambda = 0.9$ ,  $\underline{W}_0 = 0.01$ ; however these values were reduced immediately after detection of an abrupt change. The change detection algorithm used a window size,  $N$ , of 10 and a threshold  $T_\epsilon$  of 20.

Fig. 7 shows the true value and both the MSLS and RLS estimate of the  $a_{41}$  term,  $\partial P / \partial w$ , during the course of the maneuver. Fig. 8 shows just the MSLS estimate and truth on a larger scale to provide more detail.

To investigate the performance of the estimation algorithm during an impairment, a sudden tail impairment was introduced at 4.0 sec. Fig. 9 shows the results of MSLS and RLS parameter identification of the  $b_{43}$  term,  $\partial P / \partial \delta_{dt}$ , during the same maneuver.

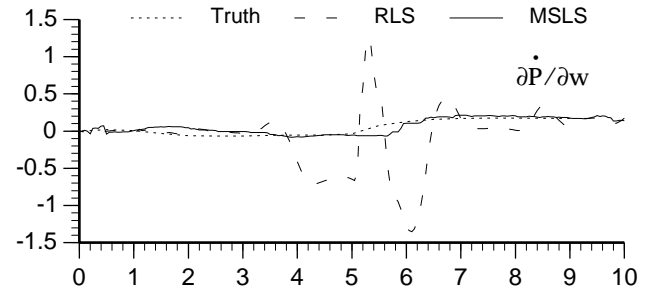


Fig. 7: MSLS vs. RLS  $\partial P / \partial w$  Parameter ID during Roll Doublet

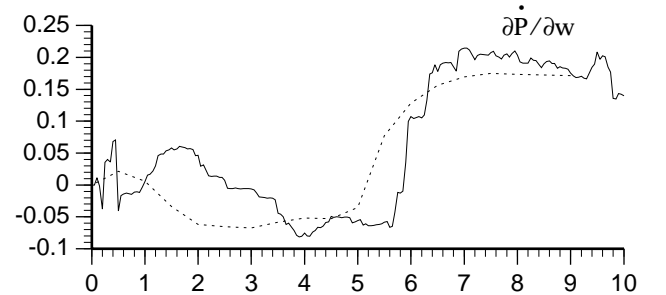


Fig. 8: Detail of MSLS  $\partial P / \partial w$  Parameter ID during Roll Doublet

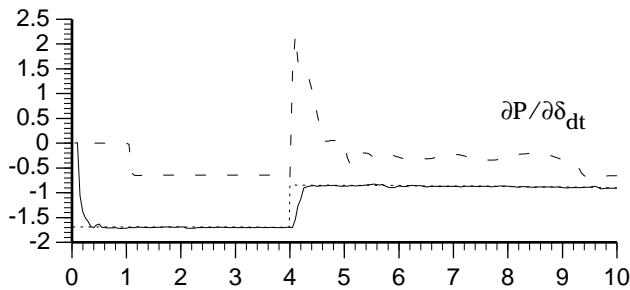


Fig. 9: MSLS vs. RLS  $\partial P / \partial \delta_{dt}$  Parameter ID during Roll Doublet (Fault at 4.0 sec.)

## Conclusions

This paper has demonstrated several ways in which the recursive least squares (RLS) algorithm can be modified so that estimation performance is dramatically improved when some or all of the parameters in a system are time-varying. In both linear and nonlinear simulations, the modified algorithm yields smooth parameter estimates that track both sudden and gradual parameter changes significantly better than does a conventional RLS approach.

The improved parameter identification capability demonstrated here is essential if a self-designing and reconfigurable flight control system is to be successfully employed. On-going SDFCS research into the use of MSLS parameter identification in conjunction with linear and nonlinear optimum control strategies is providing encouraging results [11].

Further investigations should be conducted to determine if the computational efficiency of the algorithm can be improved by applying the same modifications to an estimation algorithm based on the normal equations; it may be the case that the constrained cost function and ANI alone are sufficient to avoid numerical ill-conditioning of more conventional RLS algorithms. Additionally; the relationship between the temporal cost-function constraint, introduced here, and the spatial constraints used in Mixed Regression schemes [8] should be investigated.

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